Project Set #3 - Solutions

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1. *Where are the Accreting White Dwarfs Hiding?*

   Say we have a single degenerate system in which a companion star dumps hydrogen on a 1 M_{sun} WD. Assume that the energy released from burning H to C/O (around 10 MeV / H atom) is immediately radiated away.

   (a) If the luminosity of WD can't exceed the Eddington luminosity, then

   $$L_{\mathrm{WD}} \leq L_{\mathrm{Edd}} = \frac{4\pi G M_{\mathrm{WD}}\, c}{\kappa} \sim 10^{38} \text{ erg s}^{-1} \left(\frac{M_{\mathrm{WD}}}{M_{\mathrm{sun}}}\right)\left(\frac{0.4}{\kappa}\right) \tag{1}$$

   For electron scattering and 1 M_{sun} WD : $L_{\mathrm{WD}} \leq 10^{38}$ erg s^{-1}.

   Burning H to C/O release energy ~ 10 MeV/atom $\sim 10^{-5}$ erg/atom.

   The number of H atom that is converted to C/O per second to reach L_{Edd} is

   $$\dot{N} \sim \frac{10^{38} \text{ erg s}^{-1}}{10^{-5} \text{ erg/atom}} \sim 10^{43} \text{atom s}^{-1}. \tag{2}$$

   So, the accreted mass per second to reach L_{Edd} is

   $$\dot{M} = \dot{N}\, m_p \sim 10^{43} \times 10^{-24} \text{ g/s} \sim 10^{19} \text{ g/s} \sim 10^{-7} \ M_{\mathrm{sun}} \text{ yr}^{-1}. \tag{3}$$

   Therefore, $L_{\mathrm{WD}} \leq L_{\mathrm{Edd}}$ implies $\dot{M} \leq 10^{-7} \ M_{\mathrm{sun}}$ yr^{-1}.

   Assuming the mass transfer occurs at rate $\dot{M} \sim 10^{-7} \ M_{\mathrm{sun}}$ yr^{-1}, then the time that is needed for the WD to gain mass $\sim 0.4 \ M_{\mathrm{sun}}$ yr^{-1} is

   $$t \sim \frac{0.4 \ M_{\mathrm{sun}}}{\dot{M}} \sim \frac{0.4 \ M_{\mathrm{sun}}}{10^{-7} \ M_{\mathrm{sun}} \text{ yr}^{-1}} \approx 4 \times 10^6 \text{ years} \tag{4}$$

   (b) Assume that the radiation emitted by WD has an approximately blackbody spectrum, so $L_{\mathrm{WD}} = 4\pi R_{\mathrm{WD}}{}^2 \sigma T_{\mathrm{WD}}{}^4$. The size of a 1 M_{sun} WD is approximately the size of the Earth, so $R_{\mathrm{WD}} \sim 10^{-2} \ R_{\mathrm{sun}}$.

   Comparing to the Sun:

   $$\left(\frac{T_{\mathrm{WD}}}{T_{\mathrm{sun}}}\right)^4 = \left(\frac{L_{\mathrm{WD}}}{L_{\mathrm{sun}}}\right)\left(\frac{R_{\mathrm{sun}}}{R_{\mathrm{WD}}}\right)^2 \sim \frac{10^{38}}{10^{33}} \, 10^4 \sim 10^9. \tag{5}$$

   Therefore, we get $T_{\mathrm{WD}} \sim 10^6$ K.

Using Wien's Law, we get $\lambda_{\max} \approx 2.9 \times 10^{-7}$ cm. Photon with this wavelength has energy $E \sim 6.9 \times 10^{-10}$ erg or ~ 100 eV. Source radiating near this energy are called supersoft x-ray and have been suggested to be the progenitors of Type Ia supernovae.

(c) The observed rate of Type Ia supernovae in a typical galaxy is around 1 every 500 years. Assuming all of these supernovae come from accreting WDs, then the number of accreting WDs in a typical galaxy is

$$N_{\mathrm{WD}} \sim \text{rate} \times \text{time to reach the Chandrasekhar mass} \sim \frac{4 \times 10^6}{500} \sim 10^4.$$

The total luminosity emitted by supersoft x-ray sources in a typical galaxy is

$$L_{\mathrm{tot}} = N_{\mathrm{WD}} \times L_{\mathrm{WD}} \sim 10^4 \times 10^{38} \text{ erg/s} \sim 10^{42} \text{ erg/s} \sim 10^9 \ L_{\mathrm{sun}}. \tag{6}$$

(d) When we try to observe the source at frequency near the peak of its blackbody spectrum, we have to deal with absorption by circumstellar and interstellar gas. Bound-free cross section for hydrogen is

$$\sigma_{\mathrm{bf}} \approx 6 \times 10^{-18} \left(\frac{\nu_{\max}}{3 \times 10^{15} \text{ Hz}} \right)^{-3} \text{cm}^2, \text{ where } \nu_{\max} = \frac{E}{h} \sim 10^{16} \text{ Hz} \tag{7}$$

So, $\sigma_{\mathrm{bf}} \sim 10^{-19}$ cm^2. Suppose that the ISM has density of 1 cm^{-3} spread around ~ 10 kpc. Then, the optical depth is about

$$\tau \sim n\sigma s \sim 1 \times 10^{-19} \times 10^{22} \sim 10^3, \tag{8}$$

which is optically thick. If we observe at much higher frequencies of the Chandra observatory (~ 1 keV), then $\nu \sim 10^{17}$ Hz, $\sigma_{\mathrm{bf}} \sim 10^{-23}$ cm^2, and

$$\tau \sim n\sigma s \sim 1 \times 10^{-23} \times 10^{22} \sim 0.1, \tag{9}$$

which is optically thin.

(e) If we observe from 0.1 keV ($\nu_1 \sim 10^{16}$ Hz) to 10 keV ($\nu_2 \sim 10^{18}$ Hz), the total luminosity detected is $L_X = N_{\mathrm{WD}}(4\pi)(\pi R_{\mathrm{WD}}{}^2) \int_{\nu_1}^{\nu_2} B_\nu \, d\nu$.

Define f as the fraction of total bolometric luminosity to total luminosity in given band:

$$f = \frac{L_{\mathrm{tot}}}{L_X} = \frac{\int_0^\infty B_\nu \, d\nu}{\int_{\nu_1}^{\nu_2} B_\nu \, d\nu} = \frac{\sigma_B T_{\mathrm{WD}}{}^4}{\pi \int_{\nu_1}^{\nu_2} B_\nu \, d\nu}. \tag{10}$$

The denominator can be calculated to get $\int_{\nu_1}^{\nu_2} B_\nu \, d\nu \approx 1.58 \times 10^{19}$ erg s^{-1} cm^{-2}. Therefore $f \approx 1.14$ and $L_X \approx 8.8 \times 10^{41}$ erg s^{-1}.
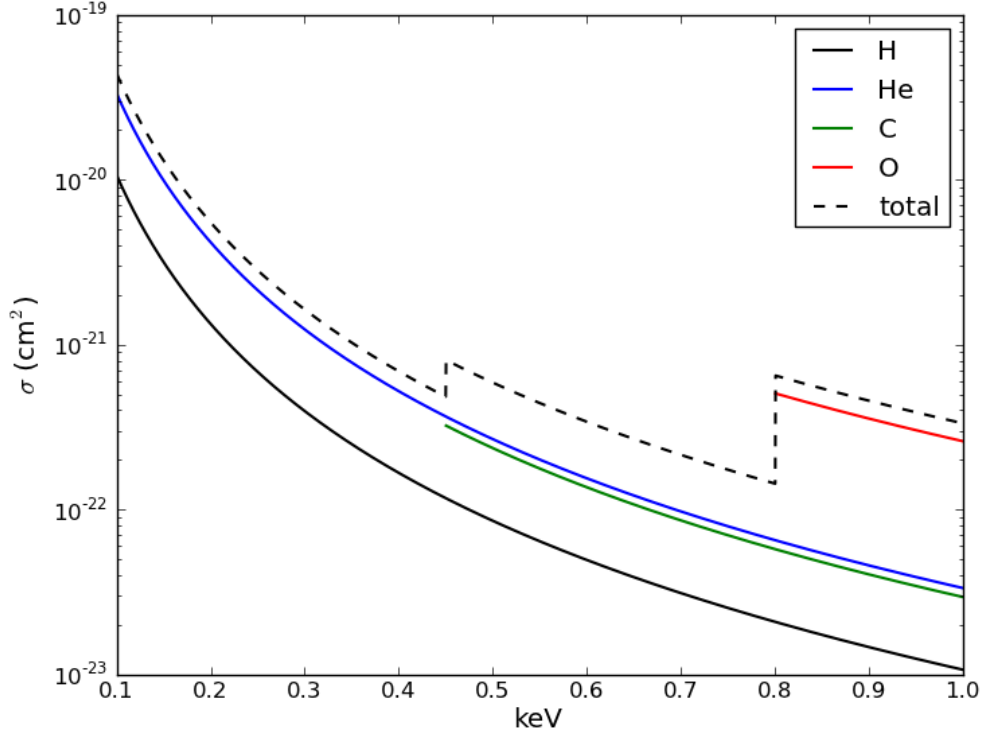
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Figure 1: Bound-free cross section for various atoms in Solar metallicity abundance.

(f) Gilfanov & Bogdan (2010) found $L_X \approx 6 \times 10^{37}$ erg s^{-1} for the bulge of M31 which has $N_{\mathrm{WD}} \approx 1.1 \times 10^3$. Since this number is an order of magnitude lower than N_{WD} calculated on part (c) then the predicted L_X for M31 bulge is $L_X \approx 8.8 \times 10^{40}$ erg s^{-1}, which is $\sim 10^3$ higher than the observed value. Therefore, requisite number of accreting WDs are not seen to explain all of SNe Ia.

(g) Assume that the progenitor system blows a wind of hydrogen with Solar metallicity which remains neutral. To calculate the bound-free cross sections in 0.1 - 10 keV, we assume the K-shell absorption dominates, so

$$\sigma_{\mathrm{bf}} \approx 6 \times 10^{-18} \, \frac{A}{Z} \left(\frac{\nu}{\nu_i}\right)^{-3} \mathrm{cm}^2, \text{ for } \nu > \nu_i. \tag{11}$$

Here we only consider the most abundant elements (H, He, C, O, and Fe), which its abundance is given on the course website. However, $\nu < \nu_i$ for iron so σ_{bf} for iron is zero. The plots are given in Figure 1.

As can be seen on the plot, the total cross section at 1 keV is about 2×10^{-22} cm^2 and it mostly contributed by cross section from oxygen.

(h) Recall that for a wind with constant mass loss rate, the density surrounding the star is given by conservation of mass:

$$\rho(r) = \frac{\dot{M}_w}{4\pi v r^2}, \tag{12}$$

where we might take the wind velocity to be comparable to the escape velocity of WD: $v \sim v_{\text{esc}} = \sqrt{2GM_{\text{WD}}/R_{\text{WD}}}$.

The optical depth due to this wind is

$$\tau = \int_{R_{\text{WD}}}^{\infty} \kappa\rho \ dr = \frac{\kappa\dot{M}_w}{4\pi v} \int_{R_{\text{WD}}}^{\infty} \frac{dr}{r^2} = \frac{\kappa\dot{M}_w}{4\pi} \sqrt{\frac{1}{2GM_{\text{WD}} \ R_{\text{WD}}}} \tag{13}$$

To attenuate the observed luminosity (in Chandra band) by a factor of ~ 20, we need optical depth $\tau \sim 3$. Also, from part (g): $\kappa = \sigma/\bar{m} \sim 100$ cm^2/g. Therefore,

$$\dot{M}_w = 4\pi \frac{\tau}{\kappa} \sqrt{2GM_{\text{WD}} \ R_{\text{WD}}} \tag{14}$$

For $M_{\text{WD}} = 1 \ M_{\text{sun}}$ and $R_{\text{WD}} = 0.01 \ R_{\text{sun}}$: $M_w = 10^{-9} M_{\text{sun}}$ yr^{-1}, which is a factor of 100 less than its accretion rate.

2. Reflection from Planetary Atmospheres.

(a) The wavelengths of the Na D and K I lines are 5890Å, 5896Å, 7665Å, and 7699Å, and so their frequencies are $c/\lambda = 5.08998 \times 10^{14}$, 5.0848×10^{14}, 3.91129×10^{14}, and 3.89401×10^{14} Hz. Their oscillator strengths are 0.64, 0.32, 1.35, and 0.68. If we take $\Gamma = \Gamma_n + \Gamma_p$, we note that $\Gamma_p = 10^{11}$ Hz is two orders of magnitude larger than Γ_n, so we can safely ignore Γ_n. Thus, the absorption profile for pressure-broadened lines is

$$\phi = \frac{\Gamma_p/(4\pi^2)}{(\nu - \nu_0)^2 + \left(\frac{\Gamma_p}{4\pi}\right)^2}. \tag{15}$$

Weighting the absorption cross-section by the oscillator strength and the abundance for each line, we obtain

$$\sigma_{\text{abs}} = \frac{\pi e^2}{m_e c}\phi(\nu) \times f \times \eta. \tag{16}$$

Here is a plot of the absorption cross section as a function of frequency for all four lines:

Now we will calculate the abundance-weighted absorption cross section in the resonances (i.e. at $\nu = \nu_0$):1.32109×10^{-18}, 6.60545×10^{-19}, 1.71694×10^{-19}, and 8.60507×10^{-20} cm^2.

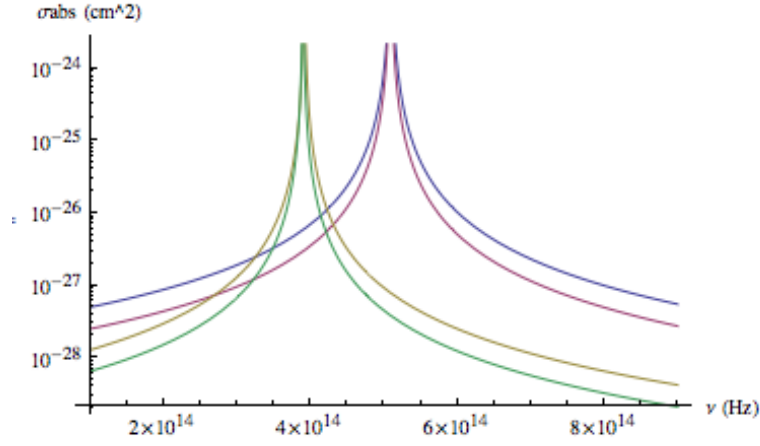(b) The Rayleigh cross-section for scattering off hydrogen, the most abundant element, is

Figure 2: Absorption cross section, weighted by abundance, for the Na I and K I resonance transitions.

$$\sigma_R = \sigma_T \left(\frac{\nu}{\nu_0}\right)^4. \tag{17}$$

where $\nu_0 = 2.5 \times 10^{15}$ is the frequency of the Lyman-α transition. We define the single scattering albedo as

$$a_{ss} = \frac{\sigma_R}{\sigma_R + \sigma_{abs}} \tag{18}$$
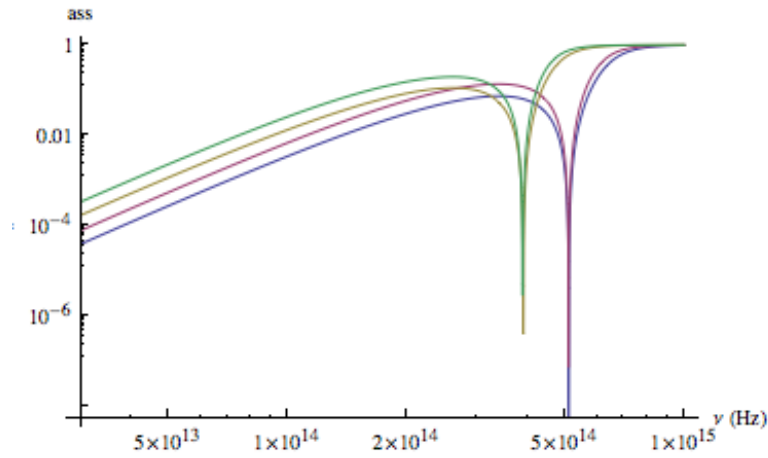
and plot it:

Figure 3: The single scattering albedo for the resonant transitions of Na I and K I. In general, the albedo rises with frequency. However, at the resonance frequencies, absorption dominates (this is the negative spike for each curve).

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